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The available information about the properties of filled elastomers under a shock-wave load is limited at present. Their behavior under normal conditions is characterized by a number of specific features [1, 2]. It is of interest to know the extent to which this specific nature manifests itself under intense pulses. Recent studies have shown, e.g., that the failure of elastomers in rarefaction waves differs from the failure of metals and polymers. When recording the velocity of the free surface of rubber upon the arrival of a shock wave Kalmykov et al. [3] and Danker et al. [4] could not detect a characteristic cleavage pulse: the measured profile monotonically decreases and differs from those postulated for the extreme cases of high and extremely low dynamic breaking strength; no clear traces of cleavage failure were detected on the preserved samples.

Qualitatively similar results in the study of cleavage in solid rocket propellants and their simulators, which are highly filled elastomers, were obtained in [5]. Recording the velocity of the free surface, Weirick [5] found an indistinct pulse of cleavage with a subsequent monotonic attenuation of the velocity. This makes it possible to determine the failure threshold, which, as noted in [3], is low: it is approximately 20 MPa for most of the compositions studied. A cleavage plate was clearly observed to form in the samples saved when the amplitude of the initial triangular compression pulse was almost an order of magnitude higher than the failure threshold. Externally, at lower pressures a preserved sample looks undamaged. The microstructure of similar compositions after shock compression was studied in [6]: at 52 MPa individual cracks inside ammonium perchlorate grains were visible in the plane of maximum tensile stress. The cracks end at the boundary with the binder. As the pressure rises the fracture of the grains grows, a cleavage plane begins to form, and a cleavage lamella separates at 145 MPa, i.e., the amplitude of the initial compression pulse, whose reflection from the free surface produces tensile stresses in the sample, should be considerably above the threshold of failure in a microsecond interval.

In view of these aspects of elastomer behavior under dynamic tension experimental studies of the behavior of elastomers in shock waves are timely. In particular, it is of interest to investigate the effect of filler-particle size on the nature of the failure. For this purpose we have studied rubbers and two highly filled elastomers with various degrees of filler dispersion under cleavage conditions.

Experimental Results. The failure of materials under cleavage conditions was studied by recording the velocity of the free surface. The experimental setup is illustrated in Fig. 1. Shock waves were generated in the sample by a striker 1, accelerated by explosion products. A shield 2 was usually placed between the sample 3 and the striker. The velocity of the free surface was recorded with a VISAR laser interferometer 5 [7] with an interferometer constant of 80.8 m/sec, making measurements to within 2 m/sec. An 8- $\mu$ m aluminum foil 4 was glued to the surface of the samples with epoxy adhesive to reflect the probe radiation. Some of the experiments were carried out with a discharge into an obstacle having low dynamic rigidity.

Rubber samples were cut from a sheet 10 mm thick. The measured densities  $\rho_0$  and sound velocity  $c_0$  under normal conditions are 1.34 g/cm<sup>3</sup> and 1.5 km/sec. The experimental results are given in Fig. 2 as the time dependence of the velocity  $w$  of the free surface. The striker thickness  $h_1$  and velocity  $w_1$ , the thickness  $h_2$  and material of the shield, and the thickness  $h_3$  of the sample for each experiment are given in Table 1 (the numbers of the experiments correspond to those of the lines in Fig. 2).

Usually after the arrival of a shock wave at the free surface of a solid the profile  $w(t)$  reveals that the velocity decreases in the incident rarefaction wave and increases upon arrival of the compressive wave (cleavage pulse) formed as a result of relaxation of the

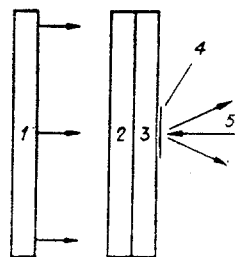


Fig. 1

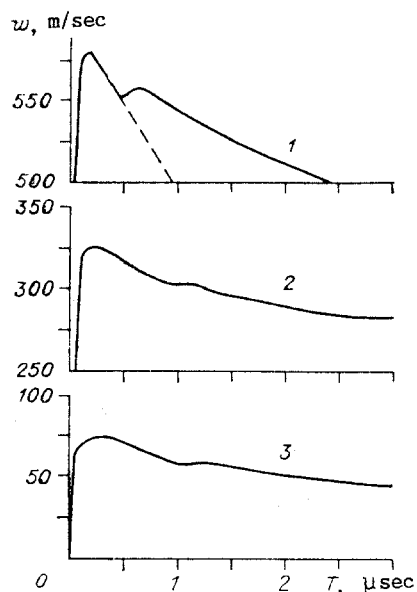


Fig. 2

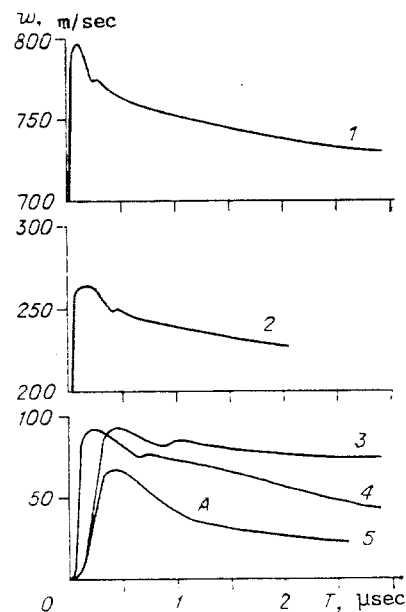


Fig. 3

tensile stresses during cleavage failure of the material. Damped oscillations of surface velocity near an average constant value are then observed, this being due to multiple reflections of water in the cleavage lamella. The velocity drop  $\Delta w$  by the arrival of the cleavage pulse front at surface is proportional to the cleavage strength  $\sigma^*$  of the material;

$$\sigma^* = 0,5\rho_0 c_0 \Delta w. \quad (1)$$

On the basis of results of experiments by the window technique we were able to calculate the profile of the free-surface velocity for experiment No. 1, on the assumption that the compressibility of the rubber is preserved in the region of negative pressure (dashed line in Fig. 2). Clearly, the measured profile should coincide with the calculated profile until the wave process is affected when the continuity of the sample is impaired by negative pressures. If the material of the sample has no appreciable breaking strength at all, the velocity of its surface should remain constant after the arrival of a shock wave. In all the experiments the profiles of the free-surface velocity are intermediate between these extreme cases. The small initial portion corresponds to the arrival of an incident load pulse at the surface and then a weak compression wave appears, similar to that recorded under cleavage. After this the surface velocity decreases slowly for a rather long time. A visual examination of the samples preserved in experiments Nos. 1 and 2 reveals no explicit signs of failure. The values of the cleavage strength of rubber, calculated from the velocity drop before the cleavage pulse, are given in Table 1.

Similar experiments were carried with a model highly filled elastomer based on butadiene-nitrile rubber. The filler content was 75% by mass, 61.6% being potassium chloride. Two types of composition were prepared: coarsely dispersed (CD) with KCl particle size 160-200  $\mu\text{m}$  and finely dispersed (FD) with particle size 20-50  $\mu\text{m}$ . The other parameters of the compositions were the same. The initial density of the samples was 1.60  $\text{g}/\text{cm}^3$  and the sound velocity under normal conditions was 1.85  $\text{km}/\text{sec}$ . The samples for the shock-wave experiments were prepared as disks with a diameter of 70-90 mm and a thickness of 4.5-5.0 mm.

The measured profiles of the free-surface velocity for the FD and MD compositions are given in Fig. 3. The corresponding load conditions and cleavage strength  $\sigma^*$ , calculated from Eq. (1), are shown in Table 2 (the numbers of the experiments correspond to those of the lines in Fig. 3). The results are qualitatively similar to those in Fig. 2 for rubber.

Experiment No. 5 was performed with a CD composition with the same arrangement as experiments Nos. 3 and 4, but by the window method: the discharge took place into hexane. Tensile stresses thus are at a comparatively shallow depth inside the sample and information about this reaches the sample-hexane interface at the time corresponding to point A. The velocity of the interface reproduces the shape of the initial compression pulse. This experiment makes it possible not only to determine the shape of the incident pulse but also to estimate the tensile stresses at which the material fails. Knowing the shock adiabat of hexane [3] and assuming that the acoustic approximation is valid for this composition, we can demonstrate

TABLE 1

No. of experiment	$h_1$ , mm	$w_1$ , m/sec	$h_2$ , mm	$h_3$ , mm	$\sigma^*$ , MPa
1	2,3	800	1,9 (PMM)	10	27
2	1,5	380	—	10	22
3	1,4	380	5,0 (Cu)	10	16

TABLE 2

No. of experiment	$h_1$ , mm	$w_1$ , m/sec	$h_2$ , mm	$h_3$ , mm	$\sigma^*$ , MPa
1	1,4	850	1,2 (PMM)	5,0 (FD)	30
2	1,8	850	5,0 (Cu)	4,8 (FD)	24
3	1,7	380	5,0 (Cu)	4,6 (CD)	15
4	1,7	380	5,0 (Cu)	4,6 (FD)	25
5	1,7	380	5,0 (Cu)	4,6 (CD)	~10

that the salient point A corresponds to tensile stresses of about 10 MPa. This value can be found only with low accuracy, mainly because of the indeterminacy of the position of point A; nevertheless, the agreement with the results of experiment No. 3 is satisfactory.

The samples were preserved in experiments Nos. 3 and 4. Clear traces of cleavage lamella forming in them were not observed, i.e., in accordance with the above-mentioned anomaly of cleavage fracture of elastomers in a microsecond interval of loading a lamella is not separated although the amplitude of the initial compression pulse substantially exceeds  $\sigma^*$ . The explanation for this is that the rupture of the elastomers is preceded by the formation of microdiscontinuities in the sample, which begins when the failure threshold is reached. The subsequent pore growth is reversible: the material can undergo considerable deformation without failing.

Discussion of the Experimental Results. The nature of the behavior of the elastomers under cleavage conditions is consistent with triaxial tension tests of vulcanizers for natural rubber [8]. In this case cavities formed at stresses of 1-3 MPa and slight strains, after which the samples underwent further deformation of several hundred percent, accompanied by an increase (with low absolute value) of the tensile stresses. After cavities are formed, the deformation conditions in their vicinity deviate from triaxial and large reversible strains become possible.

The measured cleavage strength values of 15-30 MPa can be assumed, therefore, to characterize the nucleation of microdiscontinuities in elastomers but not their failure. With the appearance of pores elastomers can have a considerable tensile strength, which manifests itself in the surface-velocity profiles as a prolonged deceleration of the layer splitting off. According to static tests the true breaking stress of a rubber rod is 88 MPa [3].

Obviously, under fixed loading conditions the deceleration should be more pronounced at a higher tensile strength. This is entirely consistent with the observed variation of the slope of the velocity profile behind the cleavage pulse in the transition from an elastomer with finely dispersed to coarsely dispersed filler (curves 3, 4 in Fig. 3). A comparison with rubber is complicated to make because of the difference in the loading conditions. The velocity gradient in the unloading part of the incident pulse is virtually the same in profiles 2 (Fig. 2) and 4 (Fig. 3). The deceleration rates are also similar. The amplitude of the compression wave in the experiment with rubber, however, is 3.5 times that in the elastomer experiment and this must be taken into account because of the possible fracture of filler particles in the compression phase [4]. As a result, we can only say that the deceleration in the rubber is slower than in the elastomer with a finely dispersed filler.

Deceleration of the layer splitting off may be caused both by the elastic component of the resistance to the reversible growth of discontinuities and by the viscosity of the material. In both cases the resistance to pore growth is inversely proportional to the pore radius. In view of this it is interesting to note that the observed rate of deceleration is also inversely proportional to the filler-particle size: it is 3.5 times higher for finely dispersed filler than for coarsely dispersed samples.

Filler-particle sizes have no obvious effect on the profile of the shock wave front (it is almost twice as strong for the CD composition). As for the shape of the minimum on the

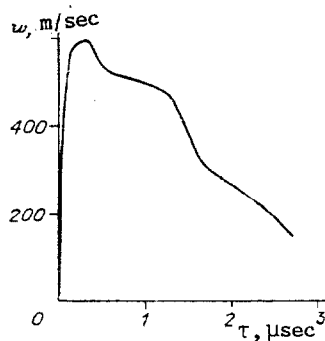


Fig. 4

velocity profile, it is flatter in experiment No. 3 (Fig. 3) than in experiment No. 4. The latter can be explained if the failure threshold of the CD composition is below 15 MPa, but the initial failure rate is low. Then in the sample that has started to fail the velocity of the free surface continues to drop with a smaller gradient until the failure rate reaches the critical value at which a minimum is formed on the  $w(t)$  profile [9]. This is also consistent with the fact that in experiment No. 5 the slope of the velocity drop to point A is greater than the average slope in experiment No. 3 until the emergence of the cleavage pulse, while the reverse should be true.

The shock adiabats of highly filled compositions were not determined, except for one point corresponding to 2.4 GPa, for which the pressure profiles in two cross sections of the sample were recorded by manganin sensors. A generalized shock adiabat [10], which is in satisfactory agreement with this experiment, was used for the pressure calculations. The resulting estimate shows that in all the experiments the amplitude of the compression waves was less than the pressure of the phase transition in KCl [11]. Anomalies associated with this thus were not observed on the velocity profiles (Fig. 3). When the pressure rises to 2 GPa or more a two-wave configuration can be expected to arise in the compression and rarefaction waves. Figure 4 shows the results of an experiment with a FD composition, using the window technique. The dependence of the interface velocity during unloading into water is shown. A compression pulse with an amplitude of 2.4 GPa was produced in the sample by a 2-mm-thick aluminum striker accelerated to 660 m/sec. A two-wave configuration is not recorded in the compression phase, probably because the sample thickness (5 mm) is too small for splitting of the shock wave to be observed. The unloading wave, however, has a pronounced two-stage configuration, which can significantly affect the nature of  $w(t)$  in the study of cleavage failure at a pressure above that of the phase transition.

In summary, the experiments indicate that the failure threshold of elastomers under cleavage conditions is 15-30 MPa. Since the failure process is so prolonged, formation of a cleavage lamella may not be observed in a microsecond interval of the shock compression even if the amplitude of initial compression pulse is an order of magnitude higher than  $\sigma^*$ . The increase in the filler-particle size not only lowers the failure threshold but also substantially reduces the deceleration rate of the free surface behind the cleavage pulse.

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RELATIONS OF THE NONLINEAR THEORY OF THREE-LAYERED SHELLS WITH  
LAYERS OF VARIABLE THICKNESS

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The versions of the theory of three-layered shells with layers of variable thickness proposed thus far have been based on various physical or kinematic hypotheses, as a rule constructed with a number of constraints on the values of the variables, the thicknesses of the layers and their variation, etc. [1-6]. The diversity of design of such shells made of ordinary materials and composites as well as their service conditions requires that re-solvents of a more general form, free of the above-mentioned constraints, be constructed.

In the work reported here we constructed the necessary complex of relations of the geometric nonlinear theory for arbitrary displacements for three-layered shells with an arbitrary geometry of external layers and filler; this complex is based on the static-kinematic model of a broken line, which has been well tested in computational practice [7]. The model used here and the corresponding equations are simplified very much to serve as a base for constructing linearized neutral equilibrium equations and formulating the corresponding problems on determining mixed [8] forms of destabilization of three-layer structural members of the class under consideration with a significant subcritical time of their stress-strain state. In particular, the general equations constructed in our work had to be applied to problems on the analysis of the stress-strain state and on determination of the critical values of acting loads for a number of aircraft structural members which have a considerably varying filler thickness and are subject to transverse bending in use (flaps, ailerons, and slats, made as three-layered plates and shells with layers of variable thickness, tail sections of the main rotor of a helicopter, etc.), as well as some problems of the engineering mechanics of three-layered structural members.

1. Problems associated with parametrizations in the noncanonical regions occupied by the layers are not trivial in the construction of a theory of three-layered shells with layers of variable thickness. These matters were investigated to various extents in [2, 3, 5, 9]. Following those studies, in order to parametrize the middle surfaces  $\sigma(k)$  ( $k = 1, 2$ ) of the outer layers of a three-layered shell as the basis for parametrization we choose the middle surface of the filler  $\sigma = \sigma(3)$ ,\* assuming that the vector equation  $\bar{r} = \bar{r}(\alpha^1)$  is given for it, and that the components  $(a_{in}, a^{in})$  and  $(b_{in}, b_i^n, b^{in})$  of the first and second metric tensors and  $(c_{in}, c^{in})$  of the discriminant tensor, the Christoffel symbols  $(\Gamma_{in}^s)$ , and other quantities determining the geometry of  $\sigma$  are specified. Using the method of normal fictitious deformation [9], we parametrize  $\sigma(k)$  in two stages, making it possible to solve this problem more correctly than in [2, 3, 5]. In the first stage we map  $\sigma$  onto the coupling (contact) surface of the layers  $\sigma(kc)$  by means of the vector equation (see Fig. 1)

$$\bar{r}_{(kc)} = \bar{r} + h_{(k)} \bar{m} \quad (h_{(k)} = \delta_{(k)} h, \quad h = t_{(3)}), \quad (1.1)$$

and in the second stage we map surfaces  $\sigma(kc)$  onto  $\sigma(k)$ , determining the radius-vector  $\bar{r}(k)$  of points  $M_{(k)} \in \sigma(k)$  by

\*The index (3), which pertains to parameters of the filler, is henceforth omitted as a rule; indices (k) and (kc) pertain to parameters on  $\sigma(k)$  and  $\sigma(kc)$ , respectively.

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